

# Matrices de proyección poblacional

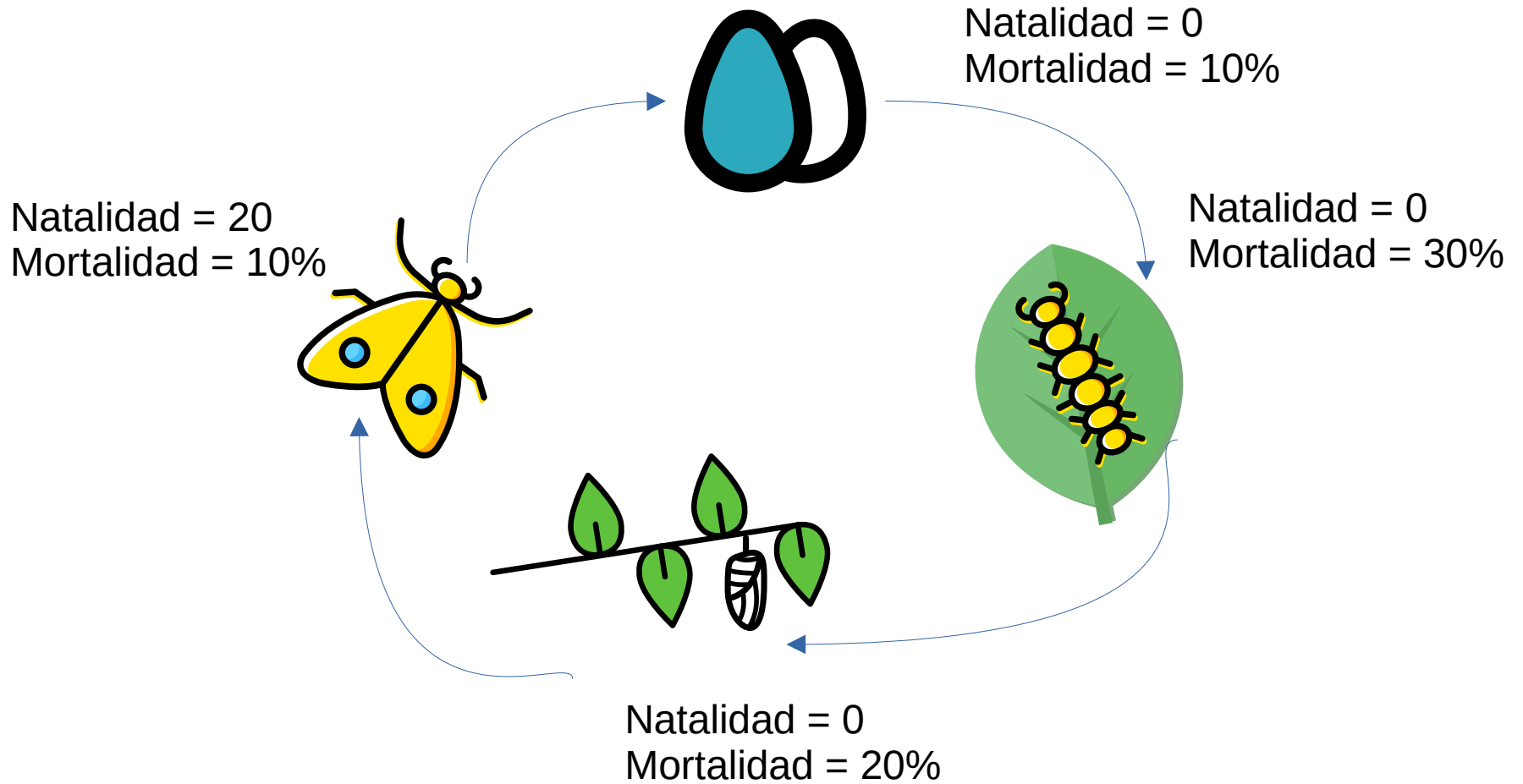
Ecología de Poblaciones  
Demografía

- Ciclos de vida → Cambios en natalidad y supervivencia
- Modelos simples como:

$$N_{t+1} = \lambda N_t$$

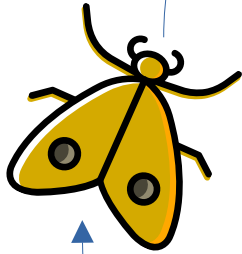
- No capturan características biológicas

# Ejemplo

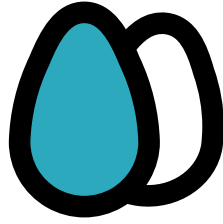


# Ejemplo

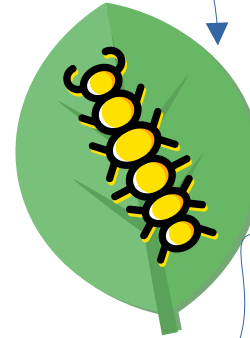
Natalidad = 10  
Mortalidad = 50%



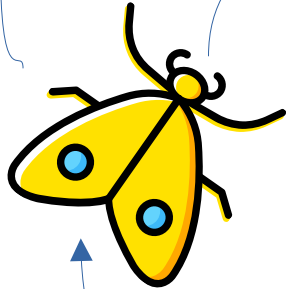
Natalidad = 0  
Mortalidad = 10%



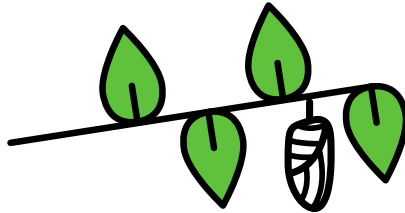
Natalidad = 0  
Mortalidad = 30%



Natalidad = 20  
Mortalidad = 10%

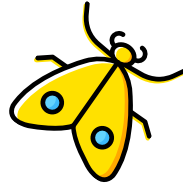
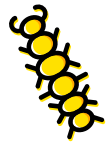


Natalidad = 0  
Mortalidad = 20%



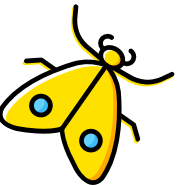
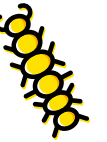
Edad	Número	$l_x$	$m_x$
0	100	0	0
1	90	0.1	0
2	72	0.3	0
3	57.6	0.2	20
4	51.84	0.1	10
5	25.92	0.5	0

# Representación como matriz

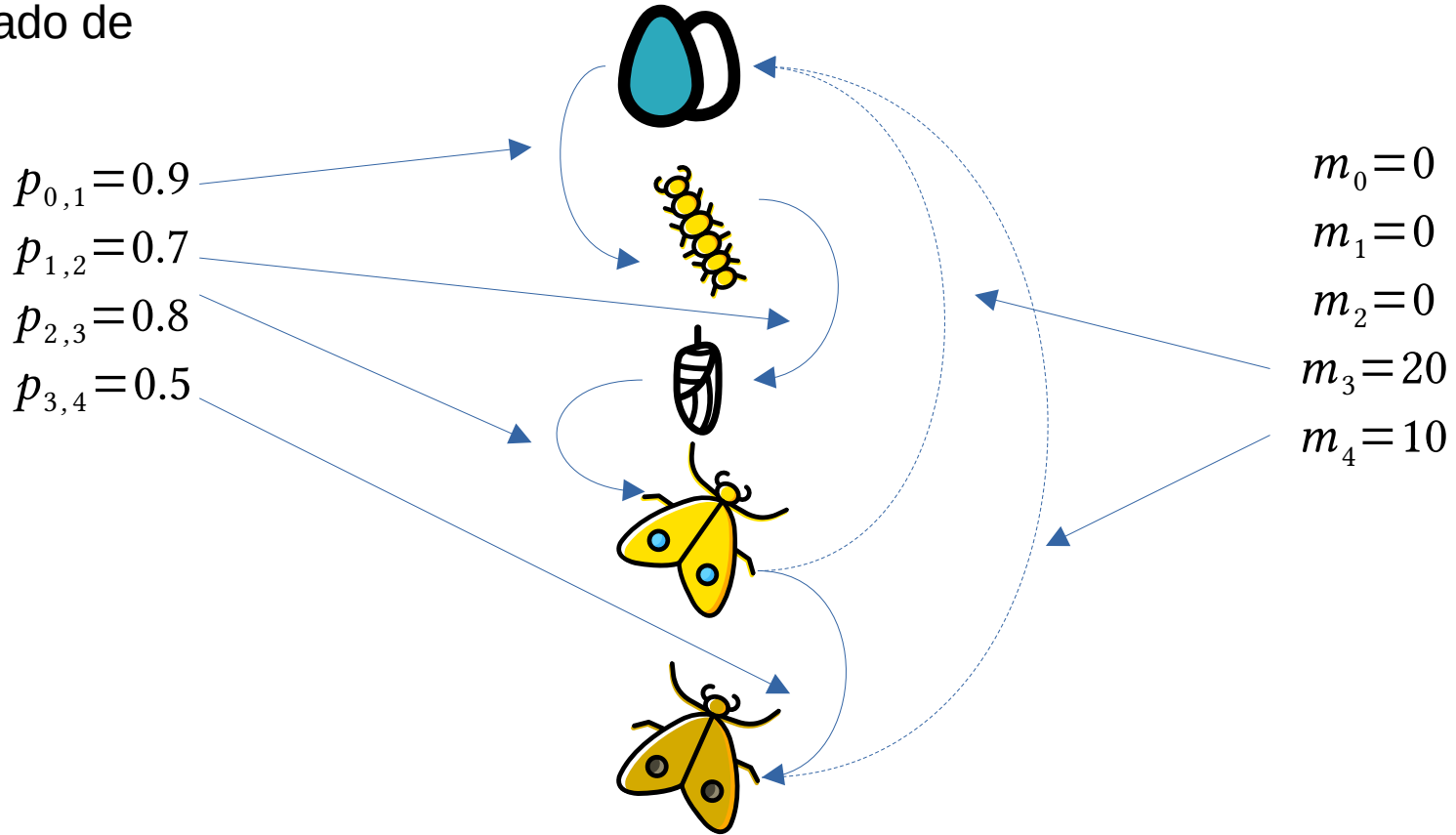


$p_i$  = Supervivencia  
 $m_i$  = Natalidad

$$\begin{bmatrix} p_{0,1} m_0 & p_{1,2} m_1 & p_{2,3} m_2 & m_3 \\ p_{0,1} & 0 & 0 & 0 \\ 0 & p_{1,2} & 0 & 0 \\ 0 & 0 & p_{2,3} & 0 \end{bmatrix} \times \begin{bmatrix} N_{0,t} \\ N_{1,t} \\ N_{2,t} \\ N_{3,t} \end{bmatrix} = \begin{bmatrix} N_{0,t+1} \\ N_{1,t+1} \\ N_{2,t+1} \\ N_{3,t+1} \end{bmatrix}$$



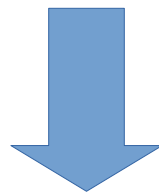
Mortalidad  
cambiante con  
edad y estado de  
desarrollo



# Solución

$$N_{0,t+1} = p_{0,1} m_0 N_{0,t} + p_{1,2} m_1 N_{1,t} + p_{2,3} m_2 N_{2,t} + p_{3,4} m_3 N_{3,t} + m_4 N_{4,t}$$

$$m_0 = 0, m_1 = 0 \text{ y } m_2 = 0$$



$$N_{0,t+1} = p_{3,4} m_3 N_{3,t} + m_4 N_{4,t}$$



- Necesitamos entonces, valores iniciales para  $N_3$  y  $N_4$

$$N_{0,t+1} = p_{3,4} m_3 N_{3,t} + m_4 N_{4,t}$$

$$N_{0,0} = 0; N_{3,0} = 10; N_{4,0} = 5$$

$$N_{0,1} = p_{3,4} m_3 \times 10 + m_4 \times 5$$

$$N_{0,1} = 0.5 \times 20 \times 10 + 10 \times 5 = 150$$

- Con una matriz completa

Diagram illustrating the calculation of predicted values from initial values using a transition matrix.

The transition matrix is shown as a 5x5 matrix:

$$\begin{bmatrix}
 0 & p_{1,2} m_1 & p_{2,3} m_2 & p_{3,4} m_3 & m_4 \\
 p_{0,1} & 0 & 0 & 0 & 0 \\
 0 & p_{1,2} & 0 & 0 & 0 \\
 0 & 0 & p_{2,3} & 0 & 0 \\
 0 & 0 & 0 & p_{3,4} & 0
 \end{bmatrix}$$

The initial values vector is:

$$\begin{bmatrix}
 N_{0,t} \\
 N_{1,t} \\
 N_{2,t} \\
 N_{3,t} \\
 N_{4,t}
 \end{bmatrix}$$

The predicted values vector is:

$$\begin{bmatrix}
 N_{0,t+1} \\
 N_{1,t+1} \\
 N_{2,t+1} \\
 N_{3,t+1} \\
 N_{4,t+1}
 \end{bmatrix}$$

The relationship is expressed as:

$$\begin{bmatrix}
 0 & p_{1,2} m_1 & p_{2,3} m_2 & p_{3,4} m_3 & m_4 \\
 p_{0,1} & 0 & 0 & 0 & 0 \\
 0 & p_{1,2} & 0 & 0 & 0 \\
 0 & 0 & p_{2,3} & 0 & 0 \\
 0 & 0 & 0 & p_{3,4} & 0
 \end{bmatrix}
 \times
 \begin{bmatrix}
 N_{0,t} \\
 N_{1,t} \\
 N_{2,t} \\
 N_{3,t} \\
 N_{4,t}
 \end{bmatrix}
 =
 \begin{bmatrix}
 N_{0,t+1} \\
 N_{1,t+1} \\
 N_{2,t+1} \\
 N_{3,t+1} \\
 N_{4,t+1}
 \end{bmatrix}$$

Labels and arrows:

- Valores iniciales** (Initial values) points to the vector  $N_{i,t}$ .
- Valores predichos** (Predicted values) points to the vector  $N_{i,t+1}$ .
- Green arrow points to the first row of the matrix.
- Blue arrow points to the second row of the matrix.
- Brown arrow points to the third row of the matrix.
- Orange arrow points to the fourth row of the matrix.
- Green arrow points to the first element of the predicted vector.
- Blue arrow points to the second element of the predicted vector.
- Brown arrow points to the third element of the predicted vector.
- Orange arrow points to the fourth element of the predicted vector.